Exam 2

You are allowed a non-graphing calculator without CAS (computer algebra system). You are also allowed both sides of a standard sized printer paper (like all the worksheets) as an equation sheet. This sheet must be handwritten (with pen or pencil). It cannot be typeset or have printed equations. Your equation sheet will be turned in with the exam, so it must have your name in the top left corner.

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1. Consider the following mini problems problems.

(a) evaluate the integral $\iint_R x^2 + y^2 \, dA$ on the upper half of a unit (i.e. radius 1) disk.

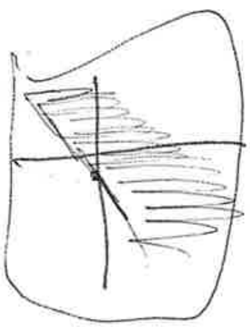
(b) Consider the line $x + 2y = 1$. Plot the inequality $x + 2y \geq 1$. Explain your reasoning

a) Polar $\int$, $x = r\cos\theta$, $y = r\sin\theta$ $\rightarrow$ $\int_0^{2\pi}\int_0^1 r^2 \, dr \, d\theta$

Inner $\int$:

$\int_0^1 r^2 \, dr = \left.\frac{r^3}{3}\right|_0^1 = \int_0^{2\pi} \frac{1}{3} \, d\theta = \left.\frac{\theta}{3}\right|_0^{2\pi} = \boxed{\frac{2\pi}{3}}$

b)



$y = \frac{-x+1}{2} \Rightarrow y = -\frac{1}{2}(\cancel{m}x+1)$

EQ of line

$y \geq -\frac{1}{2}\ldots$

y must be greater than given $-\frac{1}{2}(x+1)$

check pls?

$\nabla f = \langle 2x+1, x^2 \rangle$

2. Let $f(x, y) = x^2y + x$. Let $P = (1, 0)$. What is the equation of the tangent plane of f at P?

$z = f(x_0, y_0) + f_x(x_0, y_0) + \ldots$

1) Find partials

$f_x = 2xy + 1,$

$f_y = x^2$

② Plug P in

$f(P) = 1$

$f_x(P) = 1$

$f_y(P) = 1$

③ Plug into EQ

$\rightarrow \boxed{z = 3}$?? wth?

check 9

3. Let $f(x, y) = x^3 - x + y^2 + y$. Find all the local maxima, local minima, and saddle points of f.

Conditions

- $D > 0$
  - $f_{xx} > 0$? rel min
  - $f_{xx} < 0$? rel max
- $D < 0$ Saddle
- $D = 0$ Inc :(

For $f_{xx}(a,b) f_{yy}(a,b) - [f_{xy}(a,b)]^2$

2) Find partials!

$f_x = 3x^2 - 1$ | $f_y = 2y + 1$ | $f_{xy} = 0$

$f_{xx} = 6x$ | $f_{yy} = 2$ | $f_{yx} = 0$

1) Find a, b in EQ

$\nabla f = \langle 0, 0 \rangle \rightarrow \nabla f = \langle 3x^2 - 1, 2y + 1 \rangle$

$3x^2 = 1, x = \sqrt{\frac{1}{3}}, y = -\frac{1}{2}$

a, b

③ Plug in

$6x(2) - 0$

$12\left(\sqrt{\frac{1}{3}}\right) > 0$, and $f_{xx} > 0$ ∴

$\boxed{\text{relative minimum}}$ $\left(\sqrt{\frac{1}{3}}, -\frac{1}{2}\right)$

No saddle

No local max

4. Let $f(x, y) = x^2 - y^2$. Optimize f on the curve $x^2 + y^2 = 4$:

(a) Set up the system of equations

(b) Find solutions to the system of equations

(c) Determine which solutions correspond to maxima and which solutions correspond to minima.

$x^2 + y^2 - 4 = 0$

$\boxed{\cancel{\lambda = 0 \vee -4y}}$

Lagrange multiplier λ

(Stat)

$\nabla f = \langle 2x, -2y \rangle$ $\nabla g = \langle 2x, 2y \rangle$ pt (a)

1) $2x = 2x\lambda$

2) $-2y = 2y\lambda \Rightarrow -4y - 1 = 0$

3) $x^2 + y^2 - 4 = 0$

(b)

$\boxed{y = -1/4}$

$\boxed{\lambda = \pm 1}$

$\boxed{x = \pm\sqrt{4 + \frac{1}{4}^2}}$

~~$\lambda = 1, y = 0, x = 0$~~

~~$\Rightarrow -4, x = 1, (1, -4)$~~

~~$\nabla f = \langle 2x, -2y \rangle, \nabla g = \langle 2x, 2y \rangle$~~

~~$\Rightarrow$ ① $2x = 2x\lambda$~~ pt (a) ~~Take $\lambda = 1$~~ $x = \sqrt{-y^2 + 4}$

~~② $-2y = 2y\lambda$~~ $-2y = 2y$

~~③ $x^2 + y^2 = 4$~~

①

c) To determine, plug points from part b into constraint, ~~(1, -4)~~ $\left(\sqrt{4 + \frac{1}{4}}, -\frac{1}{4}\right)$

~~$\Rightarrow (g(1, -4)) = x^2 + y^2 = 4, \quad 1 + 16 - 4 = 13,$ maxima~~

$\boxed{\cancel{g\left(\sqrt{4 + \frac{1}{4}^2}, -\frac{1}{4}\right) \to \text{minima}}}$

$g(\sqrt{4}$

$g\left(\frac{1}{4}, -\frac{1}{4}\right) =$ min

$g(0, 0) =$ maximum

$-2y = -2y$

$-2y$ $\quad -2y + 2y = 0$

$y = -1/4$

$2x = -2x \Rightarrow 2x$

$-2x$

b $\to$ $-2y \pm 2y$ $\to$ $\boxed{0 = y \\ -4y = 0 \\ 4x = 0 \\ 0}$

$2x \pm 2x$

$y = (-1/4, 0)$

$x = (1/4, 0)$

(Ran out of ink :((

5. Consider the triangle with vertices $P_1 = (1,1)$, $P_2 = (1,2)$, and $P_3 = (2,2)$

(a) Plot this triangle

(b) Find the equations of the three sides of the triangle

in dom: ($\frac{0}{2-1} = 0$

(c) What are the x_{min}, x_{max}, y_{min}, y_{max} of the triangle.

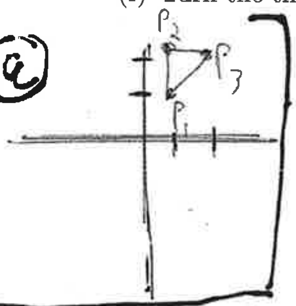
1-0-30 ??, wut

(d) What is the x-slice of the triangle

(e) What is the y-slice of the triangle

(f) Turn the the double integral into an iterated integral: $\iint_R x - y \, dA = \int \int x - y \, d\cdot \, d\cdot$

a)



d) Our x-slice comes from our lines constructed in (b), same with y-slice:

x-slice:

y-slice:

use dist func to form line $\sqrt{x^2+y^2}$

b) $P_1 \cap P_2$

$\frac{y_1 - y_2}{x_1 - x_2} \to \frac{1}{0+\sqrt{3}}$

$\boxed{y = \sqrt{3}(x-1)}$

$P_3 \cap P_2$

$2 - 2 = 0$

~~$y - 2 = 0$~~

$\boxed{y = 2}$

Line?

$P_1 \cap P_3$

~~$\sqrt{4-1+4-1} = \sqrt{6}$~~

~~$y = \sqrt{6}(x-2)$~~

$\frac{0}{2-1} = m$

$y - 1 = 0$

$\boxed{y = 1}$

c)

x min	1
x max	2
y min	1
y max	2

todo graph

6. Two half problems:

(a) What does the region given by $\frac{-1}{\sqrt{x^2+y^2}} > -\frac{1}{2}$ look like? Is it bounded? Explain your reasoning as best as you can.

(b) Show that the function $f(x, y) = |xy^2|$ is not differentiable at $(0, 0)$.

a) The region looks like some given disk-like shape which takes the natural form of $x^2+y^2=\ldots$ as a disk, in a square root $(\cdot)^{1/2}$, all $(\ldots)^{-1}$

The region is $\boxed{\text{NOT Bounded}}$ given it's $>$ and not $\geq$

$\frac{-1}{\sqrt{disk}} > -\frac{1}{2}$ or $\frac{-1}{\sqrt{disk}} + \frac{1}{2} > 0 \rightarrow$  ? but with some $\frac{-1}{\sqrt{disk}}$ form $\rightarrow$ 

b) We start by taking both partials

① $f_x(x,y) = |y^2|$ → ② We then take lim$_{x\to\infty}$ of both partials

$f_y(x,y) = |2yx|$

$\lim_{x\to 0}(|y^2|)$ or $\lim_{y\to 0}(|2yx|)$

③ when solving for ②, notice how we have 2 possible answers to come to at $(0,0)$ as we have some $\pm$ value from the $|\,|$ operator

→ Hence, because there is more than one possible solution at 0,0, we ~~can expect~~ know the function is NOT differentiable